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Perturbative QCD analysis of B to π and B to ρ transitions

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Abstract. We calculate the form factors F_0 and F_1 of $B \to \pi$ and V, A_0, A_1 and A_2 of $B \to \rho$ transition matrix elements by using the factorization formalism of perturbative QCD in the region $0 \le q^2 \le M_B^2/2$. In the limit of $m_\pi/M_B = 0$, $m_\rho/M_B = 0$, $M_b/M_B = 1$ and $(1-x) \ll 1$, the results show that F_0 and A_1 are of monopole type, V, A_0 and A_2 of dipole type, and F_1 of a combination of monopole and dipole types with dipole type dominating.

1 Introduction

CP-violation is one of the most important and mysterious phenomena in high energy physics, for which we have only the $K_L \to \pi\pi$ decay [1] and the charge asymmetry in the decay $K_L \to \pi^\pm l^\mp \nu$ [2] for more than 30 years. The mechanism of CP-violation through the complex phase of the Cabibbo-Kobayashi-Maskawa (CKM) [3] three family mixing matrix in the Weinberg-Salam model is presently the standard model for CP-violation. The B meson system offers many possibilities to investigate CP-violation [4], and the B-factories in KEK and SLAC are under construction for this purpose. In order to probe the CKM model precisely, it is crucial to obtain the values of the CKM matrix elements accurately from B meson decays. For the decays involving $b \to c$ transition, we can apply the heavy quark symmetry and it is possible to determine V_{cb} reliably through the heavy quark effective theory (HQET) [5]. However, for those involving $b \to u$ it is less likely that the heavy quark symmetry applies, and the determination of V_{ub} has heavily relied on the models for the form factors.

The dynamical content of hadron decays is described by Lorentz invariant form factors of current matrix elements. The theoretical calculation of the form factors involving $b \to u$ transition is a difficult task, since it is concerned with the nonperturbative realm of QCD and we cannot apply the heavy quark symmetry. Recently there have been active investigations of the form factors of $B \to \pi$ and $B \to \rho$ by using quark model, QCD sum rule and lattice calculations [6]. CLEO has presented first experimental results of the branching ratios of $B \to \pi l \nu$ and $B \to \rho l \nu$ [7], which are still model dependent.

In this paper we will calculate the form factors of $B \to \pi$ (F_0, F_1) and $B \to \rho$ transitions (V, A_0, A_1, A_2) by using the method which Szczepaniak et al. employed for obtaining the $B \to \pi$ form factors [8]. This method is based on the meson theory of Brodsky and Lepage [9]. [8] noticed that in the case of a heavy meson decaying into two lighter mesons the large momentum transfers are involved and the factorization formula of perturbative QCD (PQCD) for exclusive reactions becomes applicable: the amplitude can be written as a convolution of a hard-scattering quark-gluon amplitude T_h and meson distribution amplitudes $\phi(x, Q^2)$ which describe the fractional longitudinal momentum distribution amplitude of the quark and antiquark in each meson.

In the present work we obtain the q^2 dependences of the form factors in the region $0 \le q^2 \le M_B^2/2$ where the large momentum transfers are involved for the interaction between the quark and antiquark in the meson. Then we find the pole types of the form factors in the limit of $m_\pi/M_B = 0$, $m_\rho/M_B = 0$, $M_b/M_B = 1$ and $(1-x) \ll 1$. The results show that F_0 and A_1 are of monopole type, V, A_0 and A_2 of dipole type, and F_1 of a combination of monopole and dipole types with dipole type dominating. These are different from those of Wirbel et al. [10] which are assumed to be of monopole type for all form factors. Determination of the pole types of form factors are phenomenologically important. For example, the spectrum of $d\Gamma(B^0 \to \pi^- l^+ \nu)/dq^2$ is very sensitive to the pole type of F_1, and then the extraction of V_{ub} from the branching ratio $B(B^0 \to \pi^- l^+ \nu)$ is very much dependent on whether F_1 is of monopole or of dipole type.

In Sect. 2 we study the form factors of $B \to \pi$, $F_0^{B\pi}(q^2)$ and $F_1^{B\pi}(q^2)$. In Sect. 3, those of $B \to \rho$, $V^{B\rho}(q^2)$, $A_1^{B\rho}(q^2)$, $A_2^{B\rho}(q^2)$ and $A^{B\rho}(q^2)$, are calculated. We obtain in Sect. 4 the pole types of the form factors in the limit of $m_\pi/M_B = 0$, $m_\rho/M_B = 0$, $M_b/M_B = 1$ and $(1-x) \ll 1$. Sect. 5 constitutes the conclusion.

2 Form factors $F_0^{B\pi}(q^2)$ and $F_1^{B\pi}(q^2)$

From Lorentz invariance one finds the decomposition of the hadronic matrix element for $B \to \pi$ transition in terms

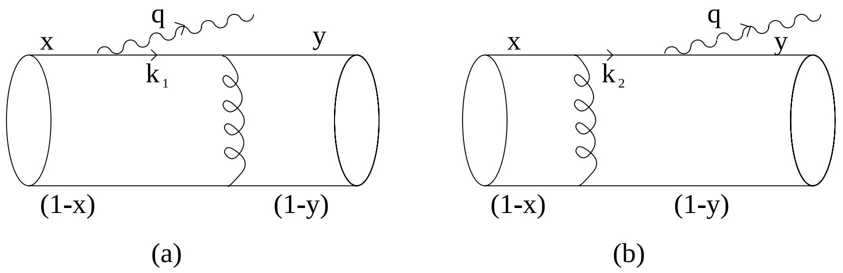

Fig. 1. Feynman diagrams to the first order in α_{s}

of hadronic form factors [10]:

$$\begin{aligned}
&\langle \pi^-(p_\pi)|V^\mu|B^0(p_B)\rangle \\
&\quad = (p_B+p_\pi)^\mu f_+^{B\pi}(q^2) + (p_B-p_\pi)^\mu f_-^{B\pi}(q^2) \\
&\quad = \left(r^\mu - \frac{m_B^2-m_\pi^2}{q^2} q^\mu\right) F_1^{B\pi}(q^2) \\
&\qquad + \frac{m_B^2-m_\pi^2}{q^2}\, q^\mu\, F_0^{B\pi}(q^2)\ , \qquad (1)
\end{aligned}$$

where $V^\mu = \bar{u}\gamma^\mu b$, $q^\mu = (p_B - p_\pi)^\mu$, $r^\mu = (p_B + p_\pi)^\mu$, and

$$\begin{aligned}
F_1^{B\pi}(q^2) &= f_+^{B\pi}(q^2)\ , \\
F_0^{B\pi}(q^2) &= f_+^{B\pi}(q^2) + \frac{q^2}{M_B^2 - m_\pi^2} f_-^{B\pi}(q^2)\ . \qquad (2)
\end{aligned}$$

In the rest frame of the decay products, F_1 and F_0 correspond to 1^- and 0^+ exchanges, respectively. At $q^2 = 0$ we have the constraint

$$F_1^{B\pi}(0) = F_0^{B\pi}(0), \qquad (3)$$

since the hadronic matrix element in (1) is nonsingular at this kinematic point.

We calculate the B to π (heavy to light) transition matrix element by using the PQCD factorization of exclusive amplitudes at high momentum transfer and neglect all final state interactions [8]. To the first order in $\alpha_{\mathrm{s}} = \alpha_{\mathrm{s}}(Q^2)$, two Feynman diagrams in Fig. 1 give the following amplitude:

$$\begin{aligned}
&\langle \pi^-(p_\pi)|V^\mu|B^0(p_B)\rangle = \frac{8\pi\alpha_{\mathrm{s}}}{3} \int_0^1 dx \int_0^{1-\epsilon} dy\, \phi_B(x) \\
&\times \left[\frac{\mathrm{Tr}\,\{(\not{p}_\pi + m_\pi)\gamma_5\gamma^\nu \not{k}_1 \gamma^\mu (\not{p}_B + g(x)M_B)\gamma_5\gamma_\nu\}}{k_1^2 Q^2} + \right. \\
&\left. \frac{\mathrm{Tr}\,\{(\not{p}_\pi + m_\pi)\gamma_5\gamma^\mu (\not{k}_2 + M_b)\gamma^\nu (\not{p}_B + g(x)M_B)\gamma_5\gamma_\nu\}}{(k_2^2 - M_b^2)Q^2} \right] \\
&\times \phi_\pi(y)\ , \qquad (4)
\end{aligned}$$

where $Q^\mu = (1-x)p_B^\mu - (1-y)p_\pi^\mu$, $k_1^\mu = -(1-x)p_B^\mu + p_\pi^\mu$, $k_2^\mu = p_B^\mu - (1-y)p_\pi^\mu$, and

$$\begin{aligned}
Q^2 &= M_B^2 \left[-(1-x)(1-y)\left(1 - \frac{q^2}{M_B^2}\right) + (1-x)^2 \right. \\
&\qquad \left. + \big((1-y)^2 - (1-x)(1-y)\big) \frac{m_\pi^2}{M_B^2} \right]\ , \\
k_1^2 &= M_B^2 \left[-(1-x)\left(1 - \frac{q^2}{M_B^2}\right) \right. \\
&\qquad \left. + (1-x)^2 + (1-(1-x)) \frac{m_\pi^2}{M_B^2} \right]\ , \\
k_2^2 - M_b^2 &= M_B^2 \left[-(1-y)\left(1 - \frac{q^2}{M_B^2}\right) + \left(1 - \frac{M_b^2}{M_B^2}\right) \right. \\
&\qquad \left. -(1-y)y \frac{m_\pi^2}{M_B^2} \right]\ . \qquad (5)
\end{aligned}$$

Here, $\bar{b}$ quark in the initial B^0 and $\bar{u}$ quark in the final π^- carry momenta xp_B and yp_π, respectively, as denoted in Fig. 1. In (4) $g(x)$ is a phenomenologically introduced parameter for B meson wave function, and $\phi_\pi(y)$ and $\phi_B(x)$ are the distribution amplitudes for π and B mesons. Our results of the q^2 dependences of the form factors will not depend on explicit forms of these distribution amplitudes. Only for the numerical estimation of the form factor values at $q^2 = 0$ we will use the following distribution amplitudes given by [8,9,11,12]

$$\begin{aligned}
\phi_\pi(x) &= \sqrt{\frac{3}{2}}\, f_\pi x(1-x), \qquad (6) \\
\phi_B(x) &= \frac{1}{2\sqrt{6}} f_B \frac{\varphi(x)}{\int_0^1 \varphi(x)dx}\ , \\
\varphi(x) &= \frac{x^2(1-x)^2}{[\epsilon^2 x + (1-x)^2]^2}\ , \qquad (7)
\end{aligned}$$

whose integrals are related to the meson decay constant by

$$\int_0^1 dx\, \phi_M(x) = \frac{1}{2\sqrt{6}}\, f_M. \qquad (8)$$

In the right hand sides of (7) and (8) there are extra factor $\frac{1}{\sqrt{2}}$ compared with [8], since in this paper we adopt the convention of the meson decay constant given by $\langle 0|A^\mu|M(p)\rangle = if_M p^\mu$ in which $f_\pi \equiv f_{\pi^+} = 131.74 \pm 0.15$ MeV [13]. In (4) we took the upper limit of the integration over momentum fraction y of a quark in the light meson as $1-\epsilon$, since the integration in the interval $1-\epsilon \le y \le 1$ corresponds to the Drell-Yan-West [14] end-point region. It gives only a small correction to the form factors, and this region is also expected to be suppressed by a Sudakov form factor [8]. Our determination of q^2 dependences of the form factors is not dependent on the exact range of integration as we will see in (41) and (42) later.

After some calculations we have

$$\langle \pi^-(p_\pi)|V^\mu|B^0(p_B)\rangle = \frac{8\pi\alpha_{\mathrm{s}}}{3} \int_0^1 dx \int_0^{1-\epsilon} dy\, \phi_B(x) \qquad (9)$$

$$\times \left[\frac{\bar{K}^a}{k_1^2 Q^2} + \frac{\bar{K}^b}{(k_2^2 - M_b^2)\, Q^2} \right] \phi_\pi(y),$$

where

$$\begin{aligned}
\bar{K}^a &= 4M_B^2 \left\{ r^\mu \left[-(1-x)\frac{q^2}{M_B^2} - x2g\frac{m_\pi}{M_B} - x\frac{m_\pi^2}{M_B^2} \right] \right. \qquad (10) \\
&\left. + q^\mu \left[(1-x)\left(2 - \frac{q^2}{M_B^2}\right) + (2-x)2g\frac{m_\pi}{M_B} - x\frac{m_\pi^2}{M_B^2} \right] \right\}, \\
\bar{K}^b &= 4M_B^2 \left\{ r^\mu \left[\left(2g\frac{M_b}{M_B} - 1\right) + (1-y)\left(1 - \frac{q^2}{M_B^2}\right) \right. \right. \\
&\left. + \left(\frac{M_b}{M_B} - y2g\right)\frac{m_\pi}{M_B} \right] + q^\mu \left[-\left(2g\frac{M_b}{M_B} - 1\right) \right. \\
&-(1-y)\left(1 - \frac{q^2}{M_B^2}\right) + \left(\frac{M_b}{M_B} - (2-y)2g\right)\frac{m_\pi}{M_B} \\
&\left.\left. -2(1-y)\frac{m_\pi^2}{M_B^2} \right] \right\}.
\end{aligned}$$

Then from (1) and (10) we have

$$\begin{aligned}
F_1^{B\pi}(q^2) &= \frac{8\pi\alpha_s}{3} \int_0^1 dx \int_0^{1-\epsilon} dy\, \phi_B(x) \\
&\quad \times \left[\frac{\bar{F}_1^a}{k_1^2 Q^2} + \frac{\bar{F}_1^b}{(k_2^2 - M_b^2) Q^2} \right] \phi_\pi(y), \qquad (11) \\
\bar{F}_1^a &= 4M_B^2 \left[-(1-x)\frac{q^2}{M_B^2} - x2g\frac{m_\pi}{M_B} - x\frac{m_\pi^2}{M_B^2} \right], \\
\bar{F}_1^b &= 4M_B^2 \left[\left(2g\frac{M_b}{M_B} - 1\right) + (1-y)\left(1 - \frac{q^2}{M_B^2}\right) \right. \\
&\quad \left. + \left(\frac{M_b}{M_B} - y2g\right)\frac{m_\pi}{M_B} \right],
\end{aligned}$$

and

$$\begin{aligned}
F_0^{B\pi}(q^2) &= \frac{8\pi\alpha_s}{3} \int_0^1 dx \int_0^{1-\epsilon} dy\, \phi_B(x) \\
&\quad \times \left[\frac{\bar{F}_0^a}{k_1^2 Q^2} + \frac{\bar{F}_0^b}{(k_2^2 - M_b^2) Q^2} \right] \phi_\pi(y), \qquad (12) \\
\bar{F}_0^a &= 4M_B^2 \left[\left[-(1-x)\frac{q^2}{M_B^2} - x2g\frac{m_\pi}{M_B} - x\frac{m_\pi^2}{M_B^2} \right] \right. \\
&\quad + \frac{q^2}{M_B^2 - m_\pi^2} \left[(1-x)\left(2 - \frac{q^2}{M_B^2}\right) \right. \\
&\quad \left.\left. + (2-x)2g\frac{m_\pi}{M_B} - x\frac{m_\pi^2}{M_B^2} \right] \right], \\
\bar{F}_0^b &= 4M_B^2 \left[\left[\left(2g\frac{M_b}{M_B} - 1\right) + (1-y)\left(1 - \frac{q^2}{M_B^2}\right) \right. \right. \\
&\quad \left. + \left(\frac{M_b}{M_B} - y2g\right)\frac{m_\pi}{M_B} \right] + \frac{q^2}{M_B^2 - m_\pi^2} \\
&\quad \left[-\left(2g\frac{M_b}{M_B} - 1\right) - (1-y)\left(1 - \frac{q^2}{M_B^2}\right) \right. \\
&\quad \left.\left. + \left(\frac{M_b}{M_B} - (2-y)2g\right)\frac{m_\pi}{M_B} - 2(1-y)\frac{m_\pi^2}{M_B^2} \right] \right].
\end{aligned}$$

For $m_\pi = 0$, we have

$$\begin{aligned}
Q^2 &= M_B^2 \left[-(1-x)(1-y)\left(1 - \frac{q^2}{M_B^2}\right) + (1-x)^2 \right], \\
k_1^2 &= M_B^2 \left[-(1-x)\left(1 - \frac{q^2}{M_B^2}\right) + (1-x)^2 \right], \\
k_2^2 - M_b^2 &= M_B^2 \left[-(1-y)\left(1 - \frac{q^2}{M_B^2}\right) + \left(1 - \frac{M_b^2}{M_B^2}\right) \right], \qquad (13)
\end{aligned}$$

$$\begin{aligned}
\bar{F}_1^a &= 4M_B^2 \left[-(1-x)\frac{q^2}{M_B^2} \right], \qquad (14) \\
\bar{F}_1^b &= 4M_B^2 \left[\left(2g\frac{M_b}{M_B} - 1\right) + (1-y)\left(1 - \frac{q^2}{M_B^2}\right) \right],
\end{aligned}$$

and

$$\begin{aligned}
\bar{F}_0^a &= 4M_B^2 \left[(1-x)\frac{q^2}{M_B^2}\left(1 - \frac{q^2}{M_B^2}\right) \right], \qquad (15) \\
\bar{F}_0^b &= 4M_B^2 \left[\left[\left(2g\frac{M_b}{M_B} - 1\right) + (1-y)\left(1 - \frac{q^2}{M_B^2}\right) \right] \right. \\
&\quad \left. \times \left(1 - \frac{q^2}{M_B^2}\right) \right].
\end{aligned}$$

Then, in the limit of $m_\pi/M_B = 0$, $M_b/M_B = 1$ and $(1-x) \ll 1$, from (11) and (12) we have

$$\begin{aligned}
F_{1,0}^{B\pi}(q^2) &= \frac{32\pi\alpha_s}{3M_B^2} \int_0^1 dx \int_0^{1-\epsilon} dy\, \phi_B(x)\, \phi_\pi(y) \\
&\quad \times \frac{1}{(1-x)(1-y)^2} f_{1,0}, \qquad (16)
\end{aligned}$$

where

$$\begin{aligned}
f_1 &= 2(1-y)\frac{1}{1 - \frac{q^2}{M_B^2}} \\
&\quad + [(2g-1) - (1-y)]\frac{1}{\left(1 - \frac{q^2}{M_B^2}\right)^2}, \qquad (17) \\
f_0 &= [(2g-1) + (1-y)]\frac{1}{1 - \frac{q^2}{M_B^2}}. \qquad (18)
\end{aligned}$$

The above results show that $F_0^{B\pi}(q^2)$ is of monopole type and $F_1^{B\pi}(q^2)$ of a combination of monopole and dipole types. The approximations $m_\pi/M_B = 0$, $m_\rho/M_B = 0$ and $M_b/M_B = 1$ are reasonable ones, since the B meson mass is much larger than the masses of light mesons or light quarks. For the B meson, it is expected that the momentum fraction of the light quark, $1-x$, is small and roughly given by $m_{\rm light}/M_B$. Therefore the distribution amplitude $\phi_B(x)$ of B meson has a sharp peak at $1-x \sim m_{\rm light}/M_B$, whose value is roughly ϵ for $\phi_B(x)$ given by (7). Then, when we integrate over x, only the range $1-x \ll 1$ contributes dominantly. For this dominant range of x, the first terms of Q^2 and k_1^2 in (13) are much larger than the second terms, $(1-x)^2$, as far as $1 - \frac{q^2}{M_B^2}$ is not very small. For this reason, we restrict the range of q^2 as $0 \le q^2 \le M_B^2/2$, and then the approximation $(1-x) \ll 1$ can be applied in the integrand safely.

From (13), Q^2 is roughly given by $-\frac{1}{2}\epsilon M_B^2(1-\frac{q^2}{M_B^2})$. Then, for $0 \le q^2 \le M_B^2/2$, the rough range of Q^2 values is between 0.38 and 0.75 GeV^2, which is still much larger than $\Lambda_{\rm QCD}^2$, and hence this is in the perturbative regime. Numerically, the $\alpha_{\rm s}(Q^2)$ is around 0.38 within 10% in this range [15].

3 Form factors $V^{B\rho}(q^2)$, $A_1^{B\rho}(q^2)$, $A_2^{B\rho}(q^2)$ and $A^{B\rho}(q^2)$

From Lorentz invariance one finds the decomposition of the hadronic matrix element for $B \to \rho$ transition in terms of hadronic form factors [10]:

$$\begin{aligned}&\left\langle \rho^-(p_\rho,\varepsilon)|(V-A)^\mu|B^0(p_B)\right\rangle \qquad (19)\\ &= \frac{2V(q^2)}{M_B+m_\rho} i\varepsilon_{\mu\alpha\beta\gamma}\varepsilon^{*\alpha}p_B^\beta p_\rho^\gamma - (M_B+m_\rho)\varepsilon^{*\mu}A_1(q^2)\\ &\quad + \frac{(\varepsilon^*\cdot p_B)}{M_B+m_\rho}(p_B+p_\rho)^\mu A_2(q^2) - 2m_\rho\frac{(\varepsilon^*\cdot p_B)}{q^2}q^\mu A(q^2).\end{aligned}$$

The form factor $A(q^2)$ can be written as

$$\begin{aligned}A(q^2) &= A_0(q^2) - A_3(q^2), \quad \text{where}\\ A_3(q^2) &= \frac{M_B+m_\rho}{2m_\rho}A_1(q^2) - \frac{M_B-m_\rho}{2m_\rho}A_2(q^2)\ ,\end{aligned} \qquad (20)$$

and at $q^2=0$ we have the constraint

$$A_0(0) = A_3(0)\ . \qquad (21)$$

We calculate the B to ρ (heavy to light) transition matrix element by using the PQCD factorization of exclusive amplitudes at high momentum transfer and neglect all final state interactions [8]. To the first order in $\alpha_{\rm s} = \alpha_{\rm s}(Q^2)$, two Feynman diagrams in Fig. 1 give the following amplitude:

$$\begin{aligned}&\left\langle \rho^-(p_\rho,\varepsilon)|V^\mu|B^0(p_B)\right\rangle = \frac{8\pi\alpha_{\rm s}}{3}\int_0^1 dx \int_0^{1-\epsilon} dy\, \phi_B(x)\\ &\times\left[\frac{\mathrm{Tr}\{(\not p_\rho + m_\rho)\not\varepsilon\gamma^\nu \not k_1\gamma^\mu(\not p_B + g(x)M_B)\gamma_5\gamma_\nu\}}{k_1^2Q^2}\right.\\ &\left. + \frac{\mathrm{Tr}\{(\not p_\rho + m_\rho)\not\varepsilon\gamma^\mu(\not k_2 + M_b)\gamma^\nu(\not p_B + g(x)M_B)\gamma_5\gamma_\nu\}}{(k_2^2-M_b^2)Q^2}\right]\\ &\times\phi_\rho(y)\ ,\end{aligned} \qquad (22)$$

where $V^\mu = \bar u\gamma^\mu b$, $Q^\mu = (1-x)p_B^\mu - (1-y)p_\rho^\mu$, $k_1^\mu = -(1-x)p_B^\mu + p_\rho^\mu$, $k_2^\mu = p_B^\mu - (1-y)p_\rho^\mu$, and Q^2, k_1^2 and $k_2^2 - M_b^2$ are given by (5) with m_π replaced by m_ρ.

In (22) $\phi_\rho(y)$ is the distribution amplitude for ρ meson. Our results of the q^2 dependences of the form factors will not depend on explicit forms of this distribution amplitudes. When we do the numerical estimation of the form factor values at $q^2=0$, we will use the following distribution amplitude given by [8,9,11,12]

$$\phi_\rho(x) = \sqrt{\frac{3}{2}}\, f_\rho x(1-x), \qquad (23)$$

where $\langle 0|V^\mu|\rho(\varepsilon)\rangle = f_\rho m_\rho \varepsilon^\mu$ in which $f_\rho \equiv f_{\rho^+} = 216\pm 5$ MeV [13]. In reality, the longitudinally and transversely polarized ρ mesons do not have the same distribution amplitude [16]. We do not take this difference into account in this work for simplicity, and the pole types of the form factors are not affected by this difference.

After some calculations we have

$$\begin{aligned}&\left\langle \rho^-(p_\rho,\varepsilon)|V^\mu|B^0(p_B)\right\rangle\\ &= \frac{8\pi\alpha_{\rm s}}{3}\int_0^1 dx\int_0^{1-\epsilon} dy\,\phi_B(x)\\ &\times\left[\frac{\bar V^a}{k_1^2Q^2} + \frac{\bar V^b}{(k_2^2-M_b^2)Q^2}\right]\phi_\rho(y)\ ,\end{aligned} \qquad (24)$$

where

$$\begin{aligned}\bar V^a &= 8M_B\frac{m_\rho}{M_B}i\varepsilon_{\mu\alpha\beta\gamma}\varepsilon^{*\alpha}p_B^\beta p_\rho^\gamma,\\ \bar V^b &= 8M_B\left(-\left(2g-\frac{M_b}{M_B}\right)-(1-y)\frac{m_\rho}{M_B}\right)\\ &\quad\times i\varepsilon_{\mu\alpha\beta\gamma}\varepsilon^{*\alpha}p_B^\beta p_\rho^\gamma\ ,\end{aligned} \qquad (25)$$

$$\begin{aligned}&\left[\frac{\bar V^a}{k_1^2Q^2} + \frac{\bar V^b}{(k_2^2-M_b^2)Q^2}\right]\\ &= 8M_Bi\varepsilon_{\mu\alpha\beta\gamma}\varepsilon^{*\alpha}p_B^\beta p_\rho^\gamma\left[\frac{1}{k_1^2Q^2}\frac{m_\rho}{M_B}\right. \qquad (26)\\ &\left.+\frac{1}{(k_2^2-M_b^2)\,Q^2}\left(-\left(2g-\frac{M_b}{M_B}\right)-(1-y)\frac{m_\rho}{M_B}\right)\right].\end{aligned}$$

To the first order in $\alpha_{\rm s} = \alpha_{\rm s}(Q^2)$ we have

$$\begin{aligned}&\left\langle \rho^-(p_\rho,\varepsilon)|A^\mu|B^0(p_B)\right\rangle = \frac{8\pi\alpha_{\rm s}}{3}\int_0^1 dx\int_0^{1-\epsilon} dy\,\phi_B(x)\\ &\times\left[\frac{\mathrm{Tr}\{(\not p_\rho + m_\rho)\not\varepsilon\gamma^\nu \not k_1\gamma^\mu\gamma_5(\not p_B + g(x)M_B)\gamma_5\gamma_\nu\}}{k_1^2Q^2}+\right.\\ &\left.\frac{\mathrm{Tr}\{(\not p_\rho + m_\rho)\not\varepsilon\gamma^\mu\gamma_5(\not k_2 + M_b)\gamma^\nu(\not p_B + g(x)M_B)\gamma_5\gamma_\nu\}}{(k_2^2-M_b^2)Q^2}\right]\\ &\times\phi_\rho(y)\ ,\end{aligned} \qquad (27)$$

where $A^\mu = \bar u\gamma^\mu\gamma_5 b$. After some calculations we have

$$\begin{aligned}&\left\langle \rho^-(p_\rho,\varepsilon)|A^\mu|B^0(p_B)\right\rangle\\ &= \frac{8\pi\alpha_{\rm s}}{3}\int_0^1 dx\int_0^{1-\epsilon} dy\phi_B(x)\\ &\times\left[\frac{\bar A^a}{k_1^2Q^2} + \frac{\bar A^b}{(k_2^2-M_b^2)Q^2}\right]\phi_\rho(y)\ ,\end{aligned} \qquad (28)$$

where

$$\bar A^a = \varepsilon^{*\mu}M_B^2 4m_\rho\left(-\left(1-\frac{q^2}{M_B^2}\right)+2(1-x)-\frac{m_\rho^2}{M_B^2}\right)$$

$$
\begin{aligned}
&+(\varepsilon^* \cdot p_B) r^\mu 4m_\rho (1-2(1-x)) \\
&+(\varepsilon^* \cdot p_B) q^\mu 4m_\rho (-1-2(1-x)) \,, \qquad (29)
\end{aligned}
$$

$$
\begin{aligned}
\bar{A}^b = \varepsilon^{*\mu} 4M_B^3 &\left[\left(\left(2g - \frac{M_b}{M_B} \right) - (1-y)\frac{m_\rho}{M_B} \right) \left(1 - \frac{q^2}{M_B^2} \right) \right. \\
&-2\left(2g\frac{M_b}{M_B} - 1 \right) \frac{m_\rho}{M_B} \\
&\left. + \left(\left(2g - \frac{M_b}{M_B} \right) - 4g(1-y) \right) \frac{m_\rho^2}{M_B^2} - (1-y)\frac{m_\rho^3}{M_B^3} \right] \\
&+(\varepsilon^* \cdot p_B) r^\mu 4M_B \left(- \left(2g - \frac{M_b}{M_B} \right) - (1-y)\frac{m_\rho}{M_B} \right) \\
&+(\varepsilon^* \cdot p_B) q^\mu 4M_B (-1) \\
&\times \left(- \left(2g - \frac{M_b}{M_B} \right) - (1-y)\frac{m_\rho}{M_B} \right) ,
\end{aligned}
$$

$$
\begin{aligned}
&\left[\frac{\bar{A}^a}{k_1^2 Q^2} + \frac{\bar{A}^b}{(k_2^2 - M_b^2) Q^2} \right] \\
&= 4M_B^3 \left\{ \varepsilon^{*\mu} \left[\frac{1}{k_1^2 Q^2} \frac{m_\rho}{M_B} \left(- \left(1 - \frac{q^2}{M_B^2} \right) \right. \right. \right. \\
&\quad \left. +2(1-x) - \frac{m_\rho^2}{M_B^2} \right) + \frac{1}{(k_2^2 - M_b^2) Q^2} \\
&\quad \times \left[\left(\left(2g - \frac{M_b}{M_B} \right) - (1-y)\frac{m_\rho}{M_B} \right) \left(1 - \frac{q^2}{M_B^2} \right) \right. \\
&\quad -2\left(2g\frac{M_b}{M_B} - 1 \right) \frac{m_\rho}{M_B} + \left(\left(2g - \frac{M_b}{M_B} \right) \right. \\
&\quad \left. \left. - 4g(1-y) \right) \frac{m_\rho^2}{M_B^2} - (1-y)\frac{m_\rho^3}{M_B^3} \right] \Bigg] + \frac{(\varepsilon^* \cdot p_B) r^\mu}{M_B^2} \\
&\quad \times \left[\frac{1}{k_1^2 Q^2} \frac{m_\rho}{M_B} (1-2(1-x)) + \frac{1}{(k_2^2 - M_b^2) Q^2} \right. \\
&\quad \left. \times \left(- \left(2g - \frac{M_b}{M_B} \right) - (1-y)\frac{m_\rho}{M_B} \right) \right] + \frac{(\varepsilon^* \cdot p_B)\, q^\mu}{M_B^2} \\
&\quad \times \left[\frac{1}{k_1^2 Q^2} \frac{m_\rho}{M_B} (-1-2(1-x)) + \frac{1}{(k_2^2 - M_b^2) Q^2} (-1) \right. \\
&\quad \left. \left. \times \left(- \left(2g - \frac{M_b}{M_B} \right) - (1-y)\frac{m_\rho}{M_B} \right) \right] \right\} . \qquad (30)
\end{aligned}
$$

For $m_\rho = 0$, Q^2, k_1^2 and $k_2^2 - M_b^2$ are given by (13), and we have

$$
\begin{aligned}
&\left[\frac{\bar{V}^a}{k_1^2 Q^2} + \frac{\bar{V}^b}{(k_2^2 - M_b^2) Q^2} \right] = \frac{\bar{V}^b}{(k_2^2 - M_b^2) Q^2} \qquad (31) \\
&= 8M_B i\varepsilon_{\mu\alpha\beta\gamma} \varepsilon^{*\alpha} p_B^\beta p_\rho^\gamma \frac{1}{(k_2^2 - M_b^2)\, Q^2} \left(2g - \frac{M_b}{M_B} \right) (-1) \,,
\end{aligned}
$$

and

$$
\begin{aligned}
&\left[\frac{\bar{A}^a}{k_1^2 Q^2} + \frac{\bar{A}^b}{(k_2^2 - M_b^2) Q^2} \right] = \frac{\bar{A}^b}{(k_2^2 - M_b^2) Q^2} \\
&= 4M_B^3 \frac{1}{(k_2^2 - M_b^2)\, Q^2} \left(2g - \frac{M_b}{M_B} \right) \qquad (32) \\
&\quad \times \left\{ \varepsilon^{*\mu} \left(1 - \frac{q^2}{M_B^2} \right) + \frac{(\varepsilon^* \cdot p_B)\, r^\mu}{M_B^2} (-1) + \frac{(\varepsilon^* \cdot p_B)\, q^\mu}{M_B^2} \right\} .
\end{aligned}
$$

4 Relations among form factors in the limit of $m_\pi/M_B = 0$, $m_\rho/M_B = 0$, $M_b/M_B = 1$ and $(1-x) \ll 1$

In this section we study the form factors $F_0^{B\pi}(q^2)$ and $F_1^{B\pi}(q^2)$ of $B \to \pi$, and $V^{B\rho}(q^2)$, $A_0^{B\rho}(q^2)$, $A_1^{B\rho}(q^2)$ and $A_2^{B\rho}(q^2)$ of $B \to \rho$, in the limit of $m_\pi/M_B = 0$, $m_\rho/M_B = 0$, $M_b/M_B = 1$ and $(1-x) \ll 1$. The approximations $m_\pi/M_B = 0$, $m_\rho/M_B = 0$ and $M_b/M_B = 1$ are reasonable ones, since the B meson mass is much larger than the masses of light mesons or light quarks. $(1-x) \ll 1$ is also a good approximation in the region $0 \le q^2 \le M_B^2/2$ as can be seen from (5), since $(1-x)$ is roughly given by the ratio of light and b quark masses or roughly by the value of the parameter ϵ in the B meson distribution amplitude (7).

From (16)–(19) and (32)–(33), we can organize the form factors as follows:

$$
\begin{aligned}
F_i(q^2) = \frac{32\pi\alpha_s}{3M_B^2} &\int_0^1 dx \int_0^{1-\epsilon} dy\, \phi_B(x)\, \phi_i(y) \\
&\times \frac{1}{(1-x)(1-y)^2} f_i \,, \qquad (33)
\end{aligned}
$$

where F_i=F_0,F_1,V,A_0,A_1,A_2, and $\phi_i(y) = \phi_\pi(y)$ for F_0 and F_1, and $\phi_i(y) = \phi_\rho(y)$ for V, A_0, A_1 and A_2. In (33) f_i are given by

$$ f_0 = [(2g-1) + (1-y)] \frac{1}{z} \,, \qquad (34) $$

$$ f_1 = 2(1-y)\frac{1}{z} + [(2g-1) - (1-y)] \frac{1}{z^2} \,, \qquad (35) $$

$$ v = (-1)(2g-1)\frac{1}{z^2} \,, \qquad (36) $$

$$ a_1 = (2g-1)\frac{1}{z} \,, \qquad (37) $$

$$ a_2 = (2g-1)\frac{1}{z^2} \,, \qquad (38) $$

$$ a = \frac{q^2}{2m_\rho M_B}(2g-1)\frac{1}{z^2} \,, \qquad (39) $$

where $z \equiv 1 - \frac{q^2}{M_B^2}$. By taking the terms up to the first order in m_ρ/M_B for a_1, a_2 and a in (29) and (30), we obtain from the relations (20):

$$ a_0 = (-1)[(2g-1) + (1-y)] \frac{1}{z^2} \,. \qquad (40) $$

The q^2 dependences of the form factors are given by (33)–(40). We obtain them in the region $0 \le q^2 \le M_B^2/2$, and present the results in Fig. 2. The formulas in (33)–(40) can be organized as

$$ F_0(q^2) = (a_\pi + b_\pi)\frac{1}{z} \,, \quad F_1(q^2) = 2b_\pi \frac{1}{z} + (a_\pi - b_\pi)\frac{1}{z^2} \,, $$

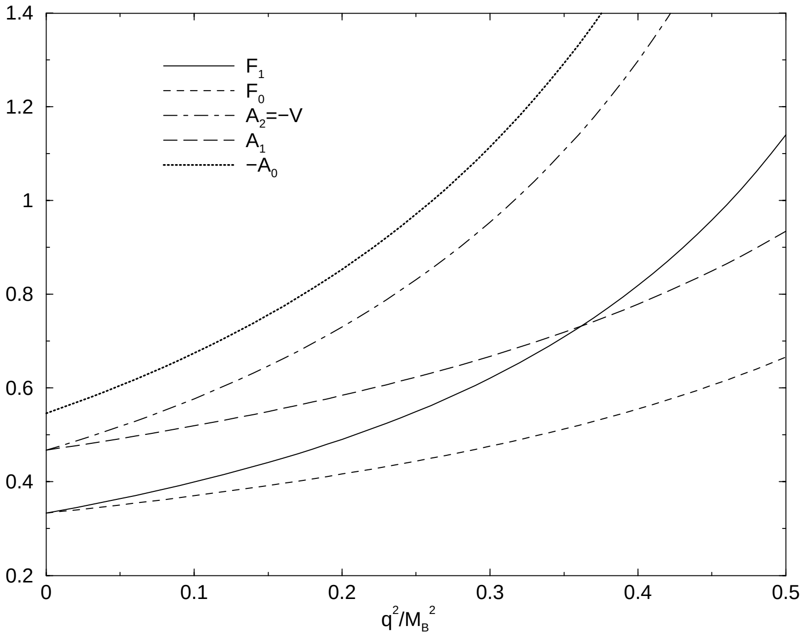

Fig. 2. The q^2 dependences of the form factors. $F_0(q^2)$ and $A_1(q^2)$ have the simple pole dependence, and $A_2(q^2)$, $V(q^2)$ and $A_0(q^2)$ have the dipole dependence. $F_1(q^2)$ has the mixture of the simple pole and dipole dependences, but the dipole dependence is dominant

$$A_1(q^2) = a_\rho \frac{1}{z} , \quad A_2(q^2) = -V(q^2) = a_\rho \frac{1}{z^2} ,$$
$$-A_0(q^2) = (a_\rho + b_\rho) \frac{1}{z^2} , \tag{41}$$

where

$$a_i = \frac{32\pi\alpha_{\mathrm{s}}}{3M_B^2} \int_0^1 dx \int_0^{1-\epsilon} dy\, \phi_B(x)\, \phi_i(y) \times \frac{2g-1}{(1-x)(1-y)^2} , \tag{42}$$
$$b_i = \frac{32\pi\alpha_{\mathrm{s}}}{3M_B^2} \int_0^1 dx \int_0^{1-\epsilon} dy\, \phi_B(x)\, \phi_i(y) \frac{1}{(1-x)(1-y)}$$

with $i = \pi$ or ρ.

We emphasize that the q^2 dependences of the form factors given in (41) are independent of the shapes of the distribution amplitudes $\phi_B(x)$, $\phi_\pi(y)$, $\phi_\rho(y)$ and the value of the parameter ϵ, as previously mentioned. Their dependences appear only in the values of the constants a_i and b_i in (42), which affect the normalizations of the form factors. From (41) we find that $F_0(q^2)$ and $A_1(q^2)$ have the simple pole q^2 dependence, and $A_2(q^2)$, $V(q^2)$ and $A_0(q^2)$ have the dipole q^2 dependence. $F_1(q^2)$ has the mixture of the simple pole and dipole q^2 dependences, but the dipole q^2 dependence is dominant. Our main results of this paper is the determination of these pole types of the form factors, and these results are independent of the normalizations of the form factors.

From (41) we find the relations among the form factors:

$$F_1\left(q^2\right) = F_0\left(q^2\right)\left(2 - \frac{1}{z}\right) + 2\frac{a_\pi}{a_\rho} A_1\left(q^2\right)\left(-1 + \frac{1}{z}\right) \tag{43}$$
$$F_0\left(q^2\right)\frac{1}{z} = -\frac{a_\pi + b_\pi}{a_\rho + b_\rho} A_0\left(q^2\right) \tag{44}$$
$$A_1\left(q^2\right)\frac{1}{z} = A_2(q^2) = -V(q^2) . \tag{45}$$

At $q^2 = 0$, (43)$-$(45) lead to the following relations:

$$F_1(0) = F_0(0) = -\frac{a_\pi + b_\pi}{a_\rho + b_\rho} A_0(0) ,$$
$$A_1(0) = A_2(0) = -V(0) . \tag{46}$$

Ball and Braun also obtained the second relation in (46) to their accuracy in their QCD sum rule calculation [17]. We calculate $F_1(0)$ and $A_1(0)$ from (41) and (42). In this calculation we took $g = 1$, $\alpha_{\mathrm{s}} = 0.38$ [8], and $f_B = 0.2$ GeV. $F_1(0)$ and $A_1(0)$ depend on the value of ϵ. The commonly used value $F_1(0) = 0.33$ obtained by Wirbel et al. [10] in quark model corresponds to $\epsilon = 0.022$, when we use the $\phi_\pi(x)$ and $\phi_B(x)$ given in (6) and (7). Then, when we use $\phi_\pi(x)$ and $\phi_\rho(x)$ given in (6) and (23) with $\epsilon = 0.022$, we obtain $a_\pi = (f_\pi/f_\rho)a_\rho = 0.28$ and $b_\pi = (f_\pi/f_\rho)b_\rho = 0.05$. These results give $A_1(0) = 0.47$ from (41), and the values of other form factors at $q^2 = 0$ can be obtained from the relations in (46).

5 Conclusion

We calculated the form factors of $B \to \pi$ and $B \to \rho$ heavy to light transition matrix elements by using the factorization formalism of perturbative QCD. We obtained the q^2 dependences of the form factors in the region $0 \le q^2 \le M_B^2/2$, since we can consider that in this region the large momentum transfers are involved for the interaction between the quark and antiquark in the meson. We found the pole types of the form factors in the limit of $m_\pi/M_B = 0$, $m_\rho/M_B = 0$, $M_b/M_B = 1$ and $(1-x) \ll 1$. These conditions are reasonable ones since the B meson mass is much larger than the masses of light mesons or light quarks, and $(1-x)$ is roughly given by the ratio of light and b quark masses.

For the heavy to heavy transitions like $B \to D^{(*)}$, HQET can be applied and all the relevant form factors are expressed by the one Isgur-Wise function [5]. However, for heavy to light transitions like $B \to \pi$ and $B \to \rho$, we cannot apply HQET, and it is very important to understand the form factors of heavy to light transitions better. Improvements in this area of study are not only invaluable for the analyses of experimental data, for example, in the extraction of the CKM matrix elements from the experimental results of the B meson decay branching ratios, but also for the better understanding of the structures of mesons. Stech studied the form factors of heavy to light transitions in the latter context [18]. In the factorization formalism of perturbative QCD we obtained the relations among the q^2 dependent form factors in (43)−(45), and the relations (46) at $q^2 = 0$. The second relaton $A_1(0) = A_2(0) = -V(0)$ in (46) was also obtained by Ball and Braun in their QCD sum rule calculation [17]. In the first relation $F_1(0) = F_0(0) = -\frac{a_\pi + b_\pi}{a_\rho + b_\rho} A_0(0)$ in (46), the first equality is a well-known relation as explained in (3), however, the second equality is not a usual one. When we use $\phi_\pi(x)$ and $\phi_\rho(x)$ given in (6) and (23), the second equality becomes $F_1(0) = -\frac{f_\pi}{f_\rho} A_0(0)$, which can be checked by measuring the differential branching ratios $d\mathcal{B}(B^0 \to \pi^- l^+ \nu)/dq^2$ and $d\mathcal{B}(B^0 \to \rho^- l^+ \nu)/dq^2$ at $q^2 = 0$, which are given by

$$\frac{d\mathcal{B}(B^0 \to \pi^- l^+ \nu)}{dq^2}|_{q^2=0} = \frac{G_F^2 M_B^5 |V_{ub}|^2}{192\pi^3 \Gamma_B} \left(1 - \frac{m_\pi^2}{M_B^2}\right)^3 |F_1(0)|^2 \, , \qquad (47)$$

$$\frac{d\mathcal{B}(B^0 \to \rho^- l^+ \nu)}{dq^2}|_{q^2=0} = \frac{G_F^2 M_B^5 |V_{ub}|^2}{192\pi^3 \Gamma_B} \left(1 - \frac{m_\rho^2}{M_B^2}\right)^3 |A_0(0)|^2 \, . \qquad (48)$$

From (47) and (48) we have

$$\frac{d\mathcal{B}(B^0 \to \pi^- l^+ \nu)/dq^2|_{q^2=0}}{d\mathcal{B}(B^0 \to \rho^- l^+ \nu)/dq^2|_{q^2=0}} = \frac{\left(1 - \frac{m_\pi^2}{M_B^2}\right)^3}{\left(1 - \frac{m_\rho^2}{M_B^2}\right)^3} \frac{|F_1(0)|^2}{|A_0(0)|^2} = 1.06 \, \frac{|F_1(0)|^2}{|A_0(0)|^2} \, . \qquad (49)$$

CLEO reported [7] $\mathcal{B}(B^0 \to \pi^- l^+ \nu) = (1.8 \pm 0.4 \pm 0.3 \pm 0.2) \times 10^{-4}$ and $\mathcal{B}(B^0 \to \rho^- l^+ \nu) = (2.5 \pm 0.4\, ^{+0.5}_{-0.7} \pm 0.5) \times 10^{-4}$. Then we expect that the ratio in the left hand side of (49) will be measured in near future, which will provide the ratio $|F_1(0)|/|A_0(0)|$.

We obtained the pole types of the form factors given by (41) and (42). We note that they are independent of the shapes of the distribution amplitudes $\phi_B(x)$, $\phi_\pi(y)$, $\phi_\rho(y)$ and the value of the parameter ϵ. Their dependences appear only in the numerical values of the constants a_i and b_i in (42), which affect the normalizations of the form factors. The formulas in (41) show that $F_0(q^2)$ and $A_1(q^2)$ have the simple pole q^2 dependence, and $A_2(q^2)$, $V(q^2)$ and $A_0(q^2)$ have the dipole q^2 dependence. $F_1(q^2)$ has the mixture of the simple pole and dipole q^2 dependences, but the dipole q^2 dependence is dominant. These results have been possible since in the case of the B meson decaying into π or ρ meson with q^2 in the range of $0 \le q^2 \le M_B^2/2$, large momentum transfers are involved, and the factorization formula of perturbative QCD for exclusive reactions becomes applicable. Therefore, the heavy to light decays possess their own characteristic and interesting properties whose deeper understandings are desirable.

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